Recall the relation between the discrete frequency, ω_k and k given by Equation 24.461 where,

$$\omega_k = \frac{2\pi k}{N} \quad 0 \le k < N \tag{5.45}$$

substituting this in Equations 5.39 and 5.40, we have,

$$H(\omega_k) = \sum_{n=0}^{N-1} h_n e^{-i\omega_k n} \tag{5.46}$$

and

$$\begin{aligned}\mathscr{P}_d^{\circ}(\omega_k) &= \frac{1}{N^2}\left|\sum_{n=0}^{N-1} h_n e^{-i\omega_k n}\right|^2 \\ &= \frac{1}{N^2}\left|\sum_{n=0}^{N-1} h_n z^{-k}\right|^2\end{aligned} \tag{5.47}$$

Comparing Equations 5.44 and 5.47, we see that Equation5.47 is a polynomial estimate of the infinite series in Equation 5.44 (not bothering with the normalization). Refer to the definition of the Laurent series expansion of an analytic function in an Annular region, Definition 24.43. Note that the z-transform may be thought of as a special case of the Laurent series of the function it transforms. Therefore, Equation 5.44 is just a Laurent series expansion. In general, the Laurent series may be approximated by a rational function with a finite number of poles and zeros. Therefore, Equation 5.42 may be approximated as,

$$\begin{aligned}H(z) &= \sum_{n=-\infty}^{\infty} h_n z^{-n} \\ &\approx \frac{G_1(z)}{G_2(z)} \\ &= \frac{\sum\limits_{\hat{n}=1}^{N} \beta_{\hat{n}} z^{-\hat{n}}}{\alpha_0 + \sum\limits_{q=1}^{Q} \alpha_q z^{-q}}\end{aligned} \tag{5.48}$$

This Equation describes the, so called, *AutoregRessive Moving Average (ARMA)* model which contains both poles and zeros.

Notice that Equation 5.46 is just the special case of the approximation given by Equation 5.48 where $G_2(z) = 1$, namely, the number of poles is 0 leaving us with an *all-zero* approximation of the Laurent series expansion of the DFT of the signal, h_n. It is conceivable that we may choose to use only poles to approximate the Laurent series and to set $G_1(z) = 1$. This leads to the *all-pole* method which will be