One of the properties, that certain classes of sets may possess, is the notion of *closure* (being closed) under certain set operations.

Definition 6.7 (Closure under a set operation). *A class, $\mathfrak{C}$, is closed under a set operation such as a union, an intersection, a complement, et cetera, if the operation as applied to its members would generate partitions which are also contained in that class.*

6.1.1 Equivalence and Partitions

Often we speak of members of a set being *equivalent*. The concept of equivalence has to be defined with respect to an *equivalence relation*. Basically, this means that every time we speak of equivalence of objects, we would have to qualify this equivalence by defining an *equivalence relation* which gives us the logic for the equivalence at hand. Consider an *equivalence relation* given by the symbol, R. Then $x \overset{\mathsf{R}}{\equiv} y$ means that x and y are equivalent as far as the logic in the equivalence relation R dictates.

Definition 6.8 (Equivalence Relation). *An equivalence relation, R, is a relation which generally allows for a binary response to the question of equivalence between objects. All equivalence relations must obey the following three properties,*

1. *Reflexivity:* $x \overset{\mathsf{R}}{\equiv} x$
2. *Symmetry:* $x \overset{\mathsf{R}}{\equiv} y \Longleftrightarrow y \overset{\mathsf{R}}{\equiv} x$
3. *Transitivity:* $x \overset{\mathsf{R}}{\equiv} y \wedge y \overset{\mathsf{R}}{\equiv} z \implies x \overset{\mathsf{R}}{\equiv} z$

Therefore, any relation that maintains the above three properties is called an equivalence relation.

Another way of looking at equivalence is the amount of indiscernibility[1] *between objects. Therefore, R is also called an indiscernibility relation [43].*

Definition 6.9 (Equivalence Class). *If we pool all the objects that are equivalent into distinct classes of objects in the universal set $\mathscr{X}$, each distinct class containing only equivalent objects is called an equivalence class and is denoted by $[\xi]_{\mathsf{R}}$ for equivalence relation R. A formal mathematical definition of $[\xi]_{\mathsf{R}}$ is as follows,*

$$\mathscr{X}_\xi = [\xi]_{\mathsf{R}} \tag{6.3}$$

$$\overset{\Delta}{=} \{x \in \mathscr{X} : x \overset{\mathsf{R}}{\equiv} \xi\} \tag{6.4}$$

[1] *Indiscernible* means “not distinct”, hence *indiscernible* objects are objects that are *similar*.