Definition 6.72 (Statistic). *A statistic is any function of a sample which is independent of the sample's distribution.*

A statistic may have different purposes. Any statistic that is used to estimate a statistical parameter is called an *estimator*. For example, the *sample mean* defined by Equation 6.173 is a statistic and an estimator of the statistical parameter, the true mean, of a distribution.

An important statistic is the *sufficient statistic* associated with a random variable, X.

Definition 6.73 (Sufficient Statistic). *A sufficient statistic, $v(x)$, is a statistic defined for a random variable, $X : x \in \mathscr{X}$ which contains in itself, all the relevant information in the X.*

Definition 6.74 (Efficiency of a Statistic). *The efficiency of a statistic is the ratio of the intrinsic accuracy of its random sampling distribution to the amount of information in the data from which it has been derived.*

This definition is valid for small samples of data with any distribution and is not limited to *Normal distributions*. For *large samples*, this is the *relevant information* utilized by the statistic of interest. For large samples with an underlying *Normal (Gaussian) distribution*, if we know the *variance* of any *sufficient statistic*, then we may compute the *efficiency* of any other statistic by the following ratio,

$$E_i \triangleq \frac{\sigma_s^2}{\sigma_i^2} \tag{6.172}$$

where E_i denotes the *efficiency* of statistic s_i, σ_i^2 is the variance computed based on statistic s_i and σ_s^2 is the variance computed from any *sufficient statistic*, s_s. Note that *efficiency*, E_i, is the fraction of *relevant information* utilized by the statistic of interest, s_i (for large samples).

Definition 6.75 (Statistical Efficiency Criterion). *The efficiency criterion requires that the variance of a statistic weighed by the number of samples used for computing the statistic approaches the smallest possible value for the underlying distribution.*

This is apparent from the special case related to *large number of samples*, given in Equation 6.172.

Definition 6.76 (Efficient Statistic). *Any statistic that meets the statistical efficiency criterion is known as an efficient statistic and any such parameter estimate is an efficient estimate.*