6.9.3 Different Types of Mean

Depending on the problem of interest, there have been many different concepts representing the *mean* of a set of samples. Here, we will define four different such *measures* which have different usages, depending on our objectives. For these definitions, we assume that we have observed a set of N samples of $X : x \in \mathscr{X} = \mathbb{R}$, denoted by, $\{X\}_1^N = \{x_n\}, n \in \{1, 2, \cdots, N\}$.

Definition 6.78 (Arithmetic Mean). *The arithmetic mean, $\mu_A(X_1^N)$, of a set of N samples of random variable X is given by the following equation,*

$$\mu_A(X_1^N) = \frac{1}{N} \sum_{n=1}^{N} x_n \tag{6.186}$$

The *sample mean*, estimating the true mean of X, given by Equation 6.179 is *arithmetic mean* of the set of N observed samples. In most cases, when one speaks of the mean of a sequence, the *arithmetic mean* is intended.

Definition 6.79 (Geometric Mean). *The geometric mean, $\mu_G(\{x\}_1^N)$, of a set of N samples of random variable X is given by the following equation,*

$$\mu_G(\{x\}_1^N) = \sqrt[N]{\prod_{n=1}^{N} x_n} \tag{6.187}$$

The *geometric mean* has the tendency of accentuating the effects of non-conforming members of the set. As we saw in Definition 5.5, this contrast with the *arithmetic mean* is used to define spectral flatness of a spectrum.

Definition 6.80 (Harmonic Mean). *The harmonic mean, $\mu_H(\{x\}_1^N)$, of a set of N samples of random variable X is only defined for a set of positive real numbers, $x_n > 0 \; \forall \; n \in \{1, 2, \cdots, N\}$, and it is given by the following equation,*

$$\mu_H(\{x\}_1^N) = \frac{N}{\sum_{n=1}^{N} \frac{1}{x_n}} \tag{6.188}$$

The *harmonic mean* has the property that it takes on a small value if x_n for any n is small. In this sense, it is more related to the *minimum* of the set than its *arithmetic mean*. As we shall see, the *harmonic mean* is used in the *k-harmonic means algorithm*, which is used for *unsupervised clustering* and described in Section 11.2.9.

Definition 6.81 (Quadratic Mean (Root Mean Square – RMS)). *The quadratic mean or RMS, $\mu_Q\{x\}_1^N$, of a set of N samples of random variable X is defined as,*