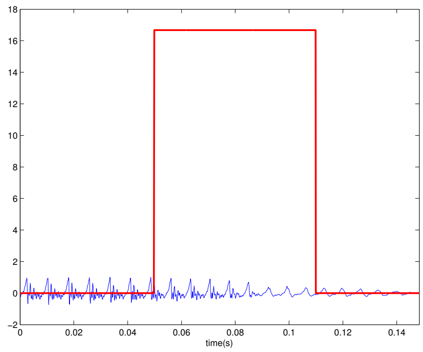

Fig. 24.19: Original signal overlaid with a square window of $60ms$ width at $t = 80ms$ with a normalized window using Equation 24.474

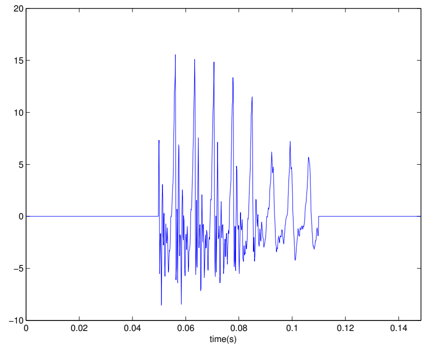

Fig. 24.20: Windowed signal with a normalized window using Equation 24.474

$$\begin{aligned} h(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{i\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(\omega,\tau)e^{i\omega t}d\omega d\tau \\ &= \int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega,\tau)e^{i\omega t}d\omega\right]d\tau \qquad (24.478) \\ &= \int_{-\infty}^{\infty} h(t)w(t-\tau)d\tau \qquad (24.479) \end{aligned}$$

The function,

$$h(t,\tau) \stackrel{\Delta}{=} h(t)w(t-\tau) \qquad (24.480)$$

is known as a *wavelet* of $h(t)$ and is given by the inverse Fourier transform of the short-time Fourier transform of $h(t)$ at time τ,

$$h(t,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega,\tau)e^{i\omega t}d\omega \qquad (24.481)$$

Therefore, since $h(t,\tau)$ is a wavelet of $h(t)$, then $H(\omega,\tau)$, the short-term Fourier transform at τ is also a *wavelet transform* of $h(t)$. We, briefly, touched upon the *wavelet series expansion* of functions in Section 24.7. Now, we have see the connection between the *Fourier transform* and the *Wavelet transform*.

In fact, one special wavelet transform, known as the *Gabor transform* is defined as a special case of short-time Fourier transform where the window function has been chosen to be a *Gaussian function*. Since

$$\int_{-\infty}^{\infty} e^{-t^2} = \sqrt{\pi} \tag{24.482}$$

the window was chosen to meet the requirement of Equation 24.476 which means,

$$w_g(t) = e^{-\pi t^2} \tag{24.483}$$

Therefore, the *Gabor transform* becomes,

$$H_G(\omega,\tau) = \int_{-\infty}^{\infty} h(t) e^{-\pi(t-\tau)^2} e^{-i\omega t} dt \tag{24.484}$$

and its inverse (known as the *Gabor wavelet*) is,

$$h_g(t,\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_g(\omega,\tau) e^{i\omega t} d\omega \tag{24.485}$$

and $h(t)$ is then given by Equation 24.479 or,

$$\begin{aligned} h(t) &= \int_{-\infty}^{\infty} h_g(t,\tau) d\tau \\ &= \int_{-\infty}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} H_g(\omega,\tau) e^{i\omega t} d\omega d\tau \end{aligned} \tag{24.486}$$

As a practical note,

$$w_G(t) \approx 0 \ \ \forall \ |t| > 2 \tag{24.487}$$

Therefore, one may write an approximation to the Gabor transform as,

$$\tilde{H}_G(\omega,\tau) = \int_{-2}^{2} h(t) e^{-\pi(t-\tau)^2} e^{-i\omega t} dt \tag{24.488}$$

where, $\tilde{H}_G(\omega,\tau) \approx H_G(\omega,\tau)$.

24.12.1 Discrete-Time Short-Time Fourier Transform DTSTFT

In this section, we will derive the *Discrete-Time Short-Time Fourier Transform (DT-STFT)* whose relationship to the continuous version is analogous to the relationship of the DTFT and the Complex Fourier Transform.

Let us start with the same objective as we did for the continuous STFT, which is the isolation of our analysis to a portion of a non-stationary signal to be able to capture local stationary effects. Consider, as did with the continuous case, a windowed version of the sampled signal, h_n at time instance m,